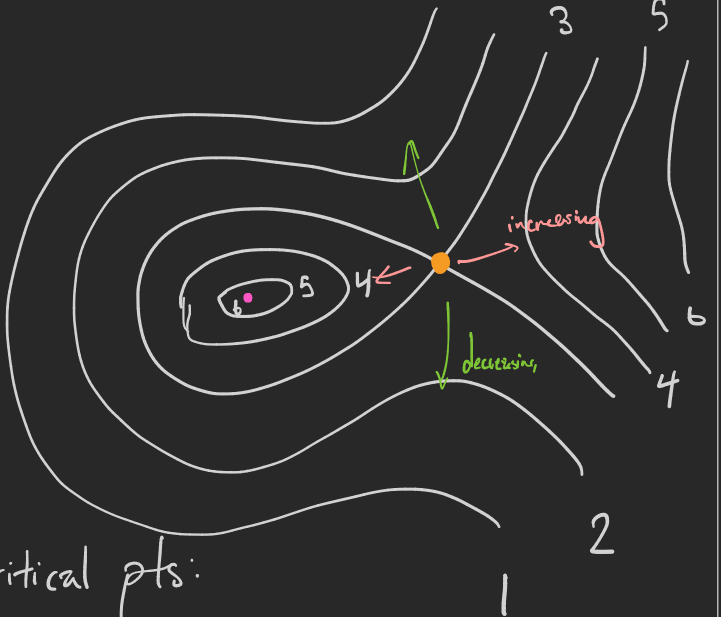
$f(x,y)$ a function whose level sets look like

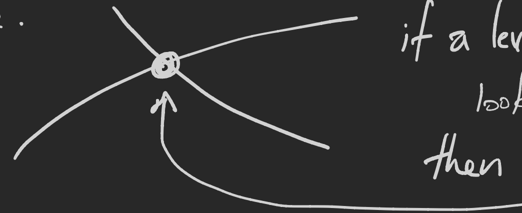


Critical pts:

- local maximum
- Can't draw any nonzero vec. perpendicular to level set, so ∇f either is zero or DNE, so this is a crit. pt.

It's a saddle pt. b/c can both inc/dec f from that point.

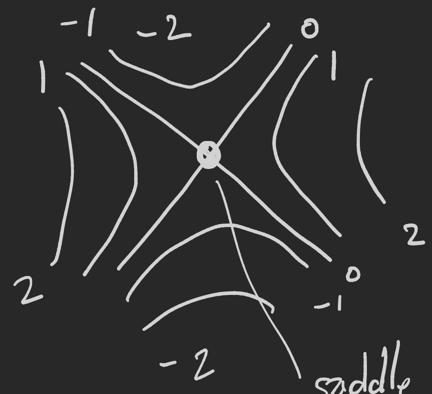
i.e.



if a level set looks like this, then this is a critical pt.

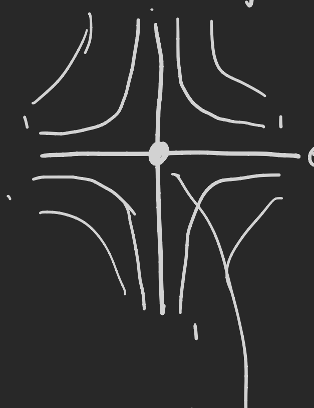
ex) (Try yourself)

$f(x,y) = x^2 - y^2$



saddle

$f(x,y) = x^2 y^2$



local min.

**) In general a "singular pt" (not smooth) of a level set is a critical pt of f.

Let $f(x, y) = x^2 y$. Suppose we want to find the tangent plane to the graph of this function when $(x, y) = (3, 0)$, and that we want to use the $\nabla F(\mathbf{r}_0) \cdot (\mathbf{r} - \mathbf{r}_0) = 0$ formula. What is an appropriate choice of F?

What do we use as $\mathbf{r}_0$?

Recall: the graph of

$$f(x,y) = x^2 y$$

is the surface defined by

$$z = x^2 y.$$

The general vector form of a plane is

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$$

a normal vec for plane

$\langle x, y, z \rangle$

any pt. in the plane

∇F is orthog to level sets of F.

So: want to interpret $z = x^2 y$ as the level set of some function F:

$$F(x,y,z) = \text{constant}.$$

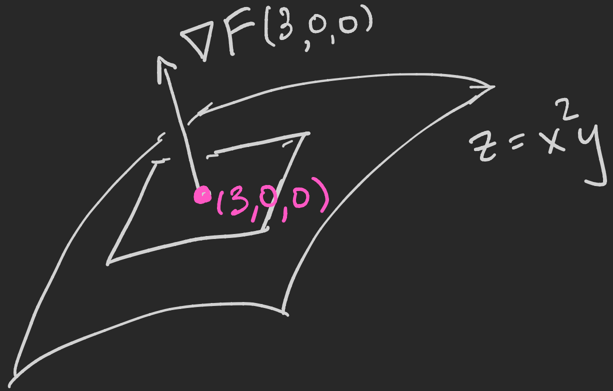
We can do e.g.

$$x^2 y - z = 0$$

$$x^2 y / z = 1$$

The second one is not so good for us since $z = 0$ @ pt. in question (but otherwise it would've been fine).

So let $F(x,y,z) = x^2 y - z$.



So tangent plane eq....

$$\nabla F(3,0,0) = \langle 0, 9, -1 \rangle$$

$$\langle 0,9,-1\rangle \cdot \left(\langle x,y,z\rangle - \langle 3,0,0\rangle\right) = 0$$

$$9y - z = 0$$

$$z = 9y.$$

Consider the plane $z = 9y$. Which of the following lines is *perpendicular* to this plane?

$x = 2, y = -3 + 9t, z = -2 + t$

$\mathbf{r}(t) = \langle 0, -2, 7\rangle + t\langle 0, 18, -2\rangle$

$$\frac{x-7}{3} = y - 2 = \frac{z+1}{9}$$

None of the above

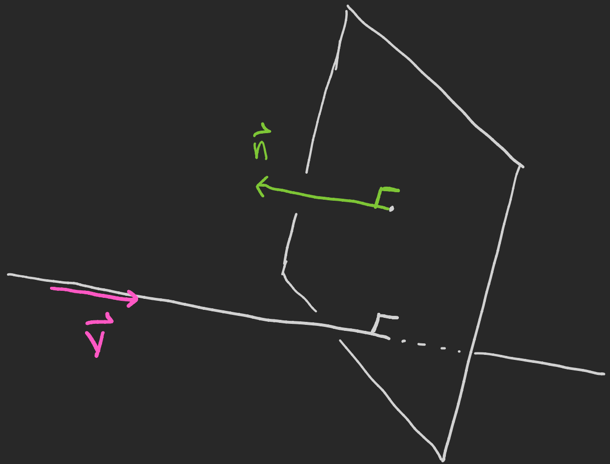
Consider the plane $z = 9y$. Which of the following lines is *parallel* to this plane?

$x = 2, y = -3 + 9t, z = -2 + t$

$\mathbf{r}(t) = \langle 0, -2, 7\rangle + t\langle 0, 18, -2\rangle$

$$\frac{x-7}{3} = y - 2 = \frac{z+1}{9}$$

None of the above



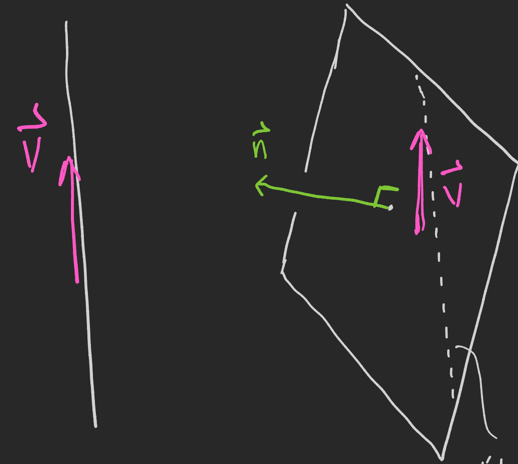
Line and plane are perpendicular when $\vec{n}$ and $\vec{v}$ are parallel.

$$z = 9y$$

$$9y - z = 0$$

$$0x + 9y + (-1)z = 0$$

$$\vec{n} = \langle 0, 9, -1 \rangle$$



"shadow" of line in plane

Line and plane are parallel when $\vec{n}$ and $\vec{v}$ orthogonal

i.e. $\vec{n} \cdot \vec{v} = 0$.

Reading off $\vec{v}$:

- $\begin{cases} x = 2 + 0t \\ y = -3 + 9t \\ z = -2 + t \end{cases}$ $\quad \vec{v} = \langle 0, 9, 1 \rangle$

- $\vec{r}(t) = \langle 0, -2, 7 \rangle + t \underbrace{\langle 0, 18, -2 \rangle}_{\vec{v}}$ ← this one is perp. to plane.

- $\dfrac{x-7}{3} = y - 2 = \dfrac{z+1}{9} = t$

$$x = 3t + 7$$
$$y = t + 2$$
$$z = 9t - 1$$

$\vec{v} = \langle 3, 1, 9 \rangle$ ← this one is parallel to plane.